sponding to a series-connected radial stub with the same actual dimensions, thereby giving significant differencies in the frequency behavior of the two structures.

Finally, the effectiveness of the method is stressed by the presentation of preliminary theoretical results obtained by an extension of the planar approach to the case of a lossy radial stub.

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# The Effects of Attenuation on the Born Reconstruction Procedure for Microwave Diffraction Tomography

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*Abstract* —**The effects of attenuation on the Born inversion process for diffraction tomography are investigated. The exact forward scattered fields for a lossy dielectric cylinder in a lossy medium are calculated and are then used to reconstruct an image of the scattering object. An accurate image is produced only when the cylinder acts as a small perturbation on the dielectric constant of the external medium. Results indicate that even small losses have a significant effect on the resolution of the image. It is possible, however, to eliminate image distortion by matching the loss tangent of the outside medium to that within the cylinder.**

## I. Introduction

Diffraction tomography provides a method of reconstructing the cross section of dielectric objects that may be useful for medical imaging of soft tissues or other applications [1]–[3]. The forward scattered fields of the object are measured for a variety of angles of incidence of a plane wave, and an inversion procedure is used to recreate the scattering centers within the object. The inversion uses approximations that are strictly valid only when the object is weakly scattering and there are no loss mechanisms. Recently, Slaney, Kak, and Larson [3] have made a computational study of the limitations of inversion processes based on the Born [4]–[6] and Rytov [7] approximations by using the exact scattered fields of a perfect uniform dielectric cylinder. They found that the accuracy of the reconstructed image is strongly determined by the dielectric constant of the cylinder relative to the uniform medium outside ($\epsilon_r$) and by its diameter relative to the wavelength of the incident electromagnetic wave.

Manuscript received July 16, 1985; revised October 28, 1985. This work was supported in part by the Australian Research Grant Scheme, Australian Telecommunications and Electronics Research Board, and University Research Grant Committee.

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IEEE Log Number 8406351.

The calculations, however, assume that there are no losses within the dielectrics although it is recognized that in the practical situation severe attenuation can occur.

In this paper, we study the effects of object and medium attenuation on diffraction tomographic reconstruction based on the Born approximation. Conditions under which distortions of the image are minimized are also investigated.

## II. Brief Outline of the Numerical Procedure

The Born approximation [1–6] assumes that the incident fields within the object are the same as when the object is absent and the object is considered to be a tenuous collection of point scatterers. Using the Greens function for spherical waves in a uniform lossless medium, a closed-form relation in the Fourier plane can be derived between the external forward scattered field and the dielectric constant distribution within the object. One way to study the effects of attenuation on this relationship is to choose a particular geometry where the exact scattered fields can be calculated when losses are included both within the object and in the uniform medium outside. Reconstruction of the distribution of scatterer using the Born approximation will then reveal the resolving power of the process.

The formulation of the boundary-value problem for a plane wave incident on a lossy dielectric cylinder with the electric field parallel to the axis of the cylinder is identical to the lossless case [8], with the exception that the external scattered fields are an infinite sum of Hankel functions of complex arguement. The major computational difficulty is to accurately evaluate these functions of large order and large complex argument.

The scattering centers within the cylinder can be derived from the external diffraction fields using the Born approximation in a manner similar to that of Slaney *et al.* [3]. The program steps are as follows.

1) Take the Fourier Transform of the scattered fields

$$E(S_x) = F[E(x)].$$

2) Multiply by the back propagation filter $f(S_x)$ [9]

$$f(S_x) = -2jk_0 m \exp(-jk_0 mR) \tag{1}$$

where

$m = \sqrt{1-(\lambda_o S_x)^2}$,

$R$ = perpendicular range of field measurements behind cylinder axis,

$k_0 = 2\pi/\lambda_0$ = wavenumber in medium outside cylinder.

3) Deposit the result onto the two-dimensional $(S_x, S_z)$ Fourier space of the object function along a semicircle of radius $1/\lambda_0$ passing through the origin.

4) Using the fact that the object spectrum must be circularly symmetric, rotate step 3) through one revolution to fill in the complete object spectrum.

5) Perform a two-dimensional inverse Fourier transform to recover the object function $k_0^2(\epsilon_r(x,z)-1)$.

Since the resolution distance of the image is $\lambda_0/2$, the computed fields are sampled at 128 points with half wavelength spacing along a line behind the cylinder. Steps 3) and 4) use bilinear interpolation to map the Fast Fourier Transform data to the regular grid on the object spectral matrix.

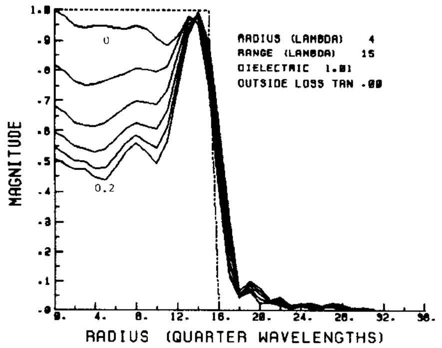


Fig. 1. Radial profile of the constructed image of a cylinder of relative dielectric constant 1.01 with respect to the outside medium and loss tangent varying from zero to 0.2 in five steps The radius of the cylinder is $4\lambda_0$ and the range at which the scattered fields are calculated is $15\lambda_0$ The medium outside the cylinder is lossless. The broken line represents the physical outline of the cylinder cross section

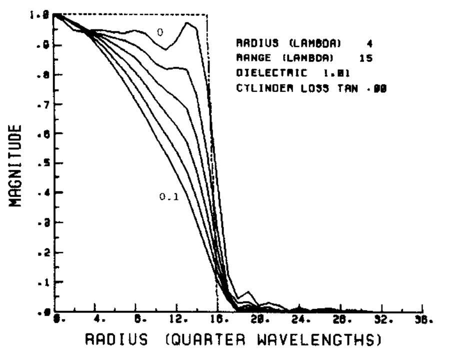


Fig. 2. Radial profile of the image of a lossless dielectric cylinder of relative dielectric constant 1.01 and radius $4\lambda_0$. The outside medium has loss tangent varying from zero to 0.1 in five steps and the scattered fields are calculated at a range of $15\lambda_0$.

## III. Results

The computer program can be run for an arbitrary set of dielectric and geometrical parameters and fails only when exponential attenuation factors reach the computational machine limit, such as when the range $R$ and the outside loss tangent are large. When no loss terms are included, the reconstructed images are essentially identical to those of Slaney *et al.* [3] for various cylinder relative dielectric constants, radius, and range. Reconstructions are accurate only when the additional phase change incurred in propagating through the cylinder is less than $\pi$. Fig. 1 shows the radial distributions of the computed object functions when the loss tangent within the cylinder is progressively increased. The loss tangent of the outside medium is zero. Clearly, the presence of cylinder losses produces a depression in the center of the image. For small loss, the amount of depression is roughly equal to the exponential loss incurred in traversing a dielectric slab with the same loss tangent as the cylinder and with thickness equal to the radius.

Fig. 2 shows the evolution of the image as the outside loss tangent is increased. The dielectric cylinder is lossless. The effect of the external attenuation is to round off the edge of the cylinder in a manner similar to a reduction in the resolving power of the reconstruction process by suppressing the high spatial-frequency content of the object spectrum. In both Figs. 1 and 2, the effect of attenuation is not significantly influenced by the range $R$ at which the fields are calculated, except for a scaling factor.

An attempt was made to modify the reconstruction algorithm to minimize the effect of attenuation in the outside medium. This might be possible since the medium is uniform and its properties

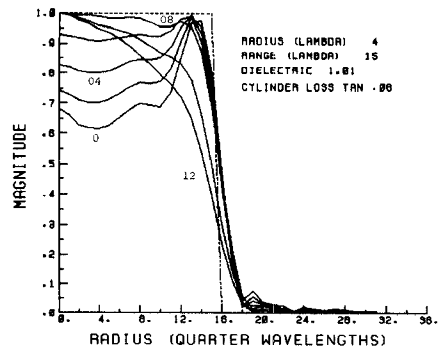


Fig. 3. Radial profile of the image of a dielectric cylinder of radius $4\lambda_0$, relative dielectric constant 1.01 and loss tangent 0 08. The loss tangent of the medium outside the cylinder is varied from zero to 0.12 in six steps

are known. Accordingly, the back propagation filter in (1) was modified to include a complex wavenumber. This approximately compensates each spatial frequency component of the scattered field for attenuation suffered in propagating a distance $R$. Because of the factor $m$, however, higher spatial frequencies are less compensated than the lower frequencies and the resulting image is smooth but shows no improvement over those of Fig. 2. Likewise, direct multiplication of the computed scattered fields by a factor to compensate for attenuation encountered in propagating the extra distance from the cylinder axis to each field point yielded only a marginal improvement in image cross section. The failure of the compensation attempts is not surprising since the reconstruction process exists because of the reversible nature of Fourier transforms that occur in the diffraction process when losses are absent. To include loss, a Laplance transform approach is required for which no simple inversion is possible.

In comparing Figs. 1 and 2, it can be seen that the presence of attenuation inside and outside the cylinder affect the reconstructed image in different regions. The effects appear to be complimentary and if the loss within the cylinder is set to a fixed value, it is possible to select an external loss that recreates an accurate image. This is demonstrated in Fig. 3, where the cylinder loss tangent is set to 0.08 and the outside loss tangent is varied. When the external loss tangent equals that of the cylinder, an optimal image is produced. Furthermore, the criteria is not influenced by the distance $R$ at which the scattered fields are determined. These results, and those of the previous section, imply that the validity of the Born approximation requires an additional constraint on the properties of the object. In other words, both the refractive index and the loss tangent of the object cannot depart significantly from those of the outside medium. Repeated calculations with a variety of parameters indicate that the effects are small when

$$\Delta(LT)\times(r/\lambda_0)\leq 0.1$$

where $\Delta(LT)$ is the difference in loss tangents between inside and outside media and $r$ is the radius of the cylinder.

## IV. Conclusion

The exact fields scattered by a lossy dielectric cylinder in a uniform lossy medium have been calculated and a tomographic inversion process based on the Born approximation has been used to reconstruct a mapping of the scattering centers within the cylinder. The procedure is valid only when the cylinder is weakly scattering so that the refractive index of the cylinder approximately matches that of the outside medium. When losses are included in the model, the reconstructed image suffers distortion which depends upon the loss in each medium. However, by matching the loss tangent of the surrounding medium with that of the cylinder an accurate image is produced. The results suggest that the reconstruction procedure is most accurate when the complex dielectric constant of the outside medium is chosen to approximate that expected within the object to be imaged.

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